

# Transient, global-in-time, convergent iterative coupling of acoustic BEM and elastic FEM

Alice Nassor, Marc Bonnet, Stéphanie Chaillat<sup>1</sup>

<sup>1</sup> POEMS, ENSTA Paris, Institut Polytechnique de Paris, 828 boulevard des Maréchaux, 91120 Palaiseau, France

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Motivated by an ongoing study in naval engineering, this work addresses a domain decomposition approach for the numerical solution of large-scale transient problems involving submerged structures subjected to pressure shock waves occurring at fast times and travelling in the surrounding fluid. The structure is to be modelled using an available finite element code, whereas a boundary element method (BEM), with the time variable treated by the convolution quadrature method (CQM) combined with the in-house fast multipole solver COFFEE in the Laplace domain [2], is applied to the fluid [6]. Given this framework, efficiency dictates that the coupled BEM-FEM problem be treated globally in time, while we seek an iterative coupling approach so as to use the FEM in non-intrusive manner. Our main contributions for this communication consist in:

- Formulation of Robin-Robin iterations for the coupled problem, extending to this problem the approach based on incoming and outgoing boundary traces proposed in [3] for frequency-domain coupled problems, and proof of their convergence.
- Mathematical solvability results (in the classical functional framework of transient PDEs [4]) for the transient coupled problem (tailored for the justification of our RR iterations and supplementing [1, 5]), as well as Robin (and Neumann) initial boundary-value problems, also explaining potential shortcomings of alternative treatments based on Neumann-Neumann iterations.
- Proof-of-concept numerical examples on transient acoustic scattering by elastic solids, where several improvements (such as the relaxation of RR iterations and an Aitken-type convergence accelerator) are assessed.

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