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Conservative and consistent iterative methods

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The presence of turbulent boundary layers is particularly challenging when modelling compressible flows. Discretizations of hyperbolic conservation laws serve as a stepping stone towards numerically capturing this phenomenon. The resulting initial value problem is often stiff and of high dimension, which necessitates the use of implicit time integration and iterative methods. Theory and practical experience suggests that the ability to discretely mimic the conservative properties of the conservation law is central to ensuring a robust and convergent scheme. Yet, while spatial discretizations and time integration methods often are designed upon this principle, iterative methods are usually not [1].

In this talk we explore and extend the results in [2] on iterative methods in the context of global and local conservation. Many commonly used methods are able to preserve the global conservation of an underlying implicit scheme. However, there are exceptions such as the Jacobi and Gauss-Seidel iterations.

Local conservation is a critical ingredient in the Lax-Wendroff theorem. This property is preserved by pseudo-time iterations as well as Newton's method, under certain restrictions on the space-time discretization. We present an extension of the Lax-Wendroff theorem that incorporates pseudo-time iterations and show that the resulting method in general is inconsistent, unless special care is taken in the choice of pseudo-time steps. This inconsistency materializes as a particular constant multiplying the spatial fluxes.

A simple technique based on the explicit Euler method can alleviate the inconsistency introduced by the pseudo-time iterations. Further, the same technique can be used to generate improved initial guesses for other iterative methods such as Newton's method. Numerical experiments indicate that the method considerably reduces the initial residual of Newton iterations at the cost of a single function evaluation. Finally, connections between pseudo-time iterations and Krylov subspace methods are explored and the role of consistency and conservation in Newton-Krylov methods are investigated.

REFERENCES

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