

Energy consistent machine learning closure model for fluid flow problems

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One of the main issues in many machine learning (ML) based large eddy simulation (LES) (*e.g.* neural network closure for the subgrid scale stresses [1]) is the assurance of long-term stability. Different solutions have been postulated, ranging from post-processing the ML model output to limit backscatter [1] to backpropagating through the solver [2]. However, these approaches still allow for an unphysical influx of energy into system.

To tackle this source of instability, we propose a model based on the energy equations for the resolved and unresolved scales. The time evolution of the energy present in the resolved scales is modelled by a PDE through local energy exchanges in addition to energy exchange with the energy present in the unresolved scales. The time evolution of the unresolved energy, on the other hand, is modelled in a global sense by an ODE. The coupling of these DEs is designed such that, in the inviscid limit, the total energy (resolved + unresolved) is conserved. Finally, these DEs are coupled to a LES with ML closure term through matching the change in energy of the LES to the prediction of the resolved energy PDE, improving accuracy/stability.

We apply this methodology to the 1D Burgers' equation with periodic boundary conditions. We show that by adding the introduced DEs the system remains stable for extended periods of time, even in the presence of strong forcing, and is capable of accurately predicting the time-evolution of the resolved energy. In addition to this we also show that the methodology is able to generalize well between different spatial domain sizes without re-training.

REFERENCES

- [1] J. Park and H. Choi, "Toward neural-network-based large eddy simulation: application to turbulent channel flow," *Journal of Fluid Mechanics*, vol. 914, p. A16, 2021.
- [2] H. Frezat, J. L. Sommer, R. Fablet, G. Balarac, and R. Lguensat, "A posteriori learning of quasi-geostrophic turbulence parametrization: an experiment on integration steps." <https://arxiv.org/abs/2111.06841>, 2021.