

A Parallel Adaptive Arc-Length Method

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Keywords: *Arc-Length Methods, Isogeometric Analysis, Paralellization, Spline Fitting*

Parallel computing is omnipresent in today’s scientific computer landscape starting at multicore processors in desktop computers up to massively parallel clusters. While domain decomposition methods have a long tradition in computational mechanics to decompose spatial problems into multiple subproblems that can be solved in parallel, advancing solution schemes for dynamics or quasi-statics are inherently serial processes. Methods like Parareal and Multigrid-reduction-in-time (MGRIT) [1] are based on a multigrid approach over the temporal domain, enabling parallelization in time. These techniques have been applied successfully in many fields of study, including cardiac fluid-structure interaction [2] or the training of neural networks [3]. For quasi-static simulations, however, there is no accumulating ‘time’ discretization error, hence an alternative approach is required.

In this talk, we will present an MGRIT-inspired approach to parallelize quasi-static computations. By using a domain parametrization of the arc-length instead of time, the multi-level error for the arc-length parametrization is formed by the load parameter and the solution norm; enabling to measure errors like in MGRIT. By applying local refinements in the arc-length parameter, this multi-level adaptive arc-length method refines solutions where the non-linearity in the load-response space is maximal, i.e. where the path is curved. To reduce the number of load steps that is needed for the computation of reference errors, a higher-dimensional spline is adaptively fitted and updated through the load-response solution space. The concept is easily extended for bifurcation problems, where multiple branches are computed in a similar parallel fashion. The results of the method are demonstrated within isogeometric structural analysis applications.

REFERENCES

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