

Discretize first, filter next – a new closure model approach

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Simulating multi-scale phenomena such as turbulent fluid flows is typically computationally very expensive. Filtering the smaller scales allows for using coarse discretizations, however, this requires closure models to account for the effects of the unresolved on the resolved scales. The common approach is to filter the continuous equations, but this gives rise to several commutator errors due to nonlinear terms, non-uniform filters, or boundary conditions. Traditional continuous closure models address the latter two types of errors by setting restrictions on the type of filter functions that can be used, e.g. requiring them to have vanishing moments [1, 2].

We propose a new approach to filtering, where the equations are discretized first and then filtered. The filtered discrete equations are obtained by applying a quadrature rule. The inverse filtering operation is then formulated as a discrete deconvolution problem. This has the advantage of circumventing several commutator errors that arise in the continuous filtering approach. Since the boundary conditions are already incorporated into the discrete equations, no further boundary treatment is necessary upon filtering.

We show that this new approach, – “discretize first” –, provides a stable framework for arbitrary non-uniform filters applied to one-dimensional linear equations with various boundary conditions; cases where classical methods produce instabilities. We also show that the discretely filtered differential operators may be inferred using an operator inference framework similar to [3], incorporating known physical properties of the discrete filters as constraints. This work serves as an initial step in the development of a new class of discrete and data-driven closure models for turbulent fluid flows.

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