

## On the invariant subspaces of deep learning-based reduced order models in MEMS

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Nonlinear modelling in structural mechanics has received an impressive boost recently thanks to the increasing availability of computational resources and applications to Micro-Electro-Mechanical-Systems (MEMS) dynamics. In order to identify the optimal MEMS geometry and operative conditions, designers cannot resort to standard simulation techniques, e.g. Finite Element (FE) Method, because of the long computational time, thus Reduced Order Models (ROMs) are used. A reliable ROM should guarantee that the underlying subspace is invariant, i.e. trajectories started on the manifold remain on it during the whole dynamics process. The computation of invariant manifolds in most systems represents an open problem. To overcome such limitations data-driven ROMs, in particular, Deep Learning-based ROMs (DL-ROMs) [1, 2] provide a general way to simulate the dynamics. Nevertheless on DL-ROMs little or no investigations were performed to check whether the identified subspace guarantees the aforementioned invariance property. In this contribution, we propose an in-depth investigation comparing the solution achieved by the DL-ROM proposed in [1] with the invariant manifold identified through the Direct Parametrization for Invariant Manifolds (DPIM)[3], referring to large scale models from MEMS industry e.g. micromirrors. In our benchmarks, we retrieve that the DL-ROM identified invariant subspaces well represent the DPIM one.

## REFERENCES

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