On mixed order approaches not working for goal oriented adaptivity in a space-time setting

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In dual weighted residual (DWR) methods [1] an auxiliary adjoint problem is solved to obtain an error estimator.
Due to Galerkin orthogonality the adjoint solution has to be represented in a higher order finite element space to get meaningful results. The natural approach of solving the adjoint problem directly in a higher order FE space has yielded good results for various types of stationary problems (p-Laplace, NSE [2], FSI, etc.). So when we started on extending the partition of unity (PU) approach for nonstationary problems in a space time setting [3], we initially tried a full mixed order approach. Namely, we solved the primal problem $dG(0)Q(1)$ space-time elements and the adjoint with $cG(1)Q(2)$. While error convergence was improved in comparison to global refinement, the effectivity of the estimator was lacking. Comparing this initial approach with different equal order approaches we found them to be the better choice both in space and time. In this walk we will present more of the less favourable mixed order results and our theories as to why this approach might not be that well suited in the space-time setting of nonstationary problems.

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