The 8th European Congress on Computational Methods in Applied Sciences and Engineering
ECCOMAS Congress 2022
5-9 June 2022, Oslo, Norway

On the generalization of gradient-based models from 1d to 3d: Curvature-dependence of phase-field modeling of brittle fracture

Patrick Kurzeja[1,*], **Kai Langenfeld**[1] **and Jörn Mosler**[1]

[1] Institute of Mechanics, TU Dortmund University, Leonhard-Euler-Str. 5, 44227 Dortmund, Germany, patrick.kurzeja@tu-dortmund.de, www.iofm.de

Keywords: *Phase-field modeling, fracture, curvature, gradient*

Phase-field models of (quasi-)brittle fracture typically incorporate a gradient term of the phase-field variable (φ) in their energy functional [1]. Stationarity of the entire functional then yields a set of governing equations that involve the second-order spatial derivative ($\Delta\varphi = \nabla \cdot \nabla\varphi$). Numerous works link this expression to a physical length scale $\sqrt{l}$ of the fracture process — which works conveniently in one dimension, see Eq. (1) and [2].

In three dimensions, however, this simple interpretation fails for a finite fracture width, e.g., for classic finite elements. Splitting the gradient of the phase-field variable into its amplitude and its direction ($\nabla\varphi = \alpha\,\mathbf{n}$) allows to distinguish two resulting governing terms, see Eq. (2). Only the first term resembles the idealized 1d interpretation. It operates in the gradient direction and is linked to the fracture width. The additional and typically unintentional term, though, is linked to the fracture curvature ($\nabla \cdot \mathbf{n}$).

	energetic contribution	gradient	governing term in stationarity condition	
1d	$\frac{l}{2}\,(\nabla\varphi \cdot \nabla\varphi)$	$\nabla\varphi = \alpha$	$\frac{l}{2}\,(\nabla\alpha)$	(1)
3d	$\frac{l}{2}\,(\nabla\varphi \cdot \nabla\varphi)$	$\nabla\varphi = \alpha\,\mathbf{n}$	$\frac{l}{2}\,(\nabla\alpha) \cdot \mathbf{n} + \frac{l}{2}\,\alpha\,(\nabla \cdot \mathbf{n})$	(2)

The present investigation aims at exploring this additional curvature effect in the phase-field equations. Numerical examples will highlight how it affects the behaviour of fracture evolution. Simulations of negligible and dominant curvature influence are presented as limit cases. The discussion finally focuses on how this curvature influence can be controlled numerically to obtain a traditional, curvature-free link to fracture mechanics.

REFERENCES

[1] J.-Y. Wu, V.P. Nguyen, C.T. Nguyen, D. Sutula, S. Sinaie and S.P.A. Bordas, *Chapter One - Phase-field modeling of fracture* in *Advances in Applied Mechanics* by eds. S.P.A. Bordas and D.S. Balint, Vol. **53**, pp. 1–183, Elsevier, 2020, https://doi.org/10.1016/bs.aams.2019.08.001.

[2] R.H.J. Peerlings, R. de Borst, W.A.M. Brekelmans and J.H.P. de Vree, *Gradient enhanced damage for quasi-brittle materials*, Int. J. Numer. Meth. Engng, Vol. **39**, pp. 3391–3493, 1996, https://doi.org/10.1002/(SICI)1097-0207(19961015)39:19<3391::AID-NME7>3.0.CO;2-D.