GENERIC based thermodynamically consistent discretisation in space and time for open thermomechanical systems

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ABSTRACT

This work deals with the Energy-Momentum-Entropy (EME) consistent discretisation of open thermoelastic systems in space and time. While Energy-Momentum (EM) consistent numerical methods are well-known for conservative mechanical systems, Romero introduced in [6] the class of thermodynamically consistent (TC) methods for coupled thermomechanical systems, which comply with the first and second law of thermodynamics. If a TC method respects the symmetries of the underlying system as well it can be viewed as an extension of EM methods to the dissipative regime and thus could be termed Energy-Momentum-Entropy consistent method.

The construction of EME methods is based on thermodynamically admissible evolution equations expressed in the GENERIC (General Equation for Non-Equilibrium Reversible-Irreversible Coupling) framework, originally proposed by Grmela and Öttinger [2]. This double generator formalism relies on the additive decomposition of the evolution equations into reversible and irreversible contributions and is independent of the specific choice for the thermodynamical state variable (see [5]). We explore the structure of the GENERIC framework using the entropy density (see e.g. [4]), the absolute temperature (see e.g. [3]) and the internal energy density, first for finite-dimensional systems in matrix-vector representation and then for infinite-dimensional systems in bracket form. The infinite-dimensional systems are discretised in space leading to finite-dimensional systems which can be expressed in the matrix-vector representation of the GENERIC where a GENERIC consistent spatial discretisation is introduced. Applying the notion of a discrete gradient [1] leads to EME consistent methods.
The presentation will also adress the impact of the choice of thermodynamical varaible on the numerical method. In particual, either the total temperature, the entropy density or the internal energy density are used as thermodynamical state variable.

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