Comparing various coupling methods for 1D Diffusion Equations with a analytical solution of Two phase Stefan Problem

Anusha Sunkisala[*]**, Konrad Simon**[†] **Jörn Behrens**[†]

[*]Department of Numerical Methods in Geosciences
Universität Hamburg
Grindelberg 5, 20144 Hamburg, Germany
e-mail: anusha.sunkisala@uni-hamburg.de

[†]Department of Numerical Methods in Geosciences
Universität Hamburg
Grindelberg 5, 20144 Hamburg, Germany
e-mail: joern.behrens@uni-hamburg.de

ABSTRACT

In earth system models (ESM) sub-components such as atmosphere, ocean, terrestrial and cryosphere systems are coupled with each other at their boundaries through couplers. The main function of a coupler is to interpolate the coupling fields and provide input to the sub-components. However, still it is unclear if this coupling strategy provides best framework for coupling climate models.

In this study, we investigate the influence of two different coupling strategies by means of a simplified one-dimensional model set-up proposed by Stefan (Stefan,1891). The Stefan Problem is a prototypical 1D model to show melting of polar ice caps and freezing of water due to the transfer of heat fluxes between ocean and ice. Here we compare our solution of coupling strategies to the analytical solution for the Stefan problem allowing for a rigorous analysis.

We consider Gills (1997) loose coupling algorithim as our first coupling strategie to couple the solid and liquid interface. Moreover, we use an explicit and implicit algorithm in which each half of the domain is solved separately with boundary conditions exchanging the information from the other domain. In these procedures, at the interface, computation of solid domain uses Dirichlet data obtained from the solution of the liquid domain, while the computation of liquid domain uses Neumann data obtained from the solution of the solid domain. As our second coupling stratagie, we employ explicit higher order derivatives to compute fluxes at the interface. This method is referred as tight coupling. These approximations use a five-point stencil in 1D.

Our results show that the solution with tight coupling looks closest to the analytical solution when compared to loose coupling algorithms. This may be due to the exchange of information on a large overlap. Our results need further analysis but indicate that different coupling strategies lead to artificial specific error in the solution.

REFERENCES

[1] Stefan, J. Über die theorie der Eisbildung, inbesondere über die Eisbildung im Polarmeer. *Annalen der Physik und Chemie.* (1891) **42**:269–286.

[2] Giles, M.B. Stability analysis of numerical interface conditions in fluid structure thermal analysis. *International Journal of Numerical methods in Fluids.* (1997) **25**:421–436.