Explicit Partitioned Methods based on Monolithic Formulations of the Coupled Problem

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ABSTRACT

Traditional explicit partitioned schemes exchange boundary conditions between subdomains and can be related to iterative solution methods for the coupled problem. As a result, these schemes may require multiple subdomain solves, acceleration techniques, or optimized transmission conditions to achieve sufficient accuracy and/or stability.

We present two families of synchronous partitioned methods [1] derived from a well-posed mixed finite element formulation of the coupled problem in which the transmission condition is enforced by a Lagrange multiplier. We transform the resulting Differential Algebraic Equation (DAE) to a Hessenberg index-1 form in which the algebraic equation defines the Lagrange multiplier as an implicit function of the states.

Using this fact we eliminate the multiplier and reduce the DAE to a system of explicit ODEs for the states. Explicit time integration both discretizes this system in time and decouples its equations. As a result, the temporal accuracy and stability of our formulation are governed solely by the accuracy and stability of the explicit scheme employed and are not subject to additional stability considerations as in traditional partitioned schemes.

We establish sufficient conditions for the formulation to be well-posed and prove that classical mortar finite elements on the interface are a stable choice for the Lagrange multiplier. We show that in this case the condition number of the Schur complement involved in the elimination of the multiplier is bounded by a constant. The talk will also present numerical examples illustrating the approach for two different interface problems.

REFERENCES

[1] K. Peterson, P. Bochev, and P. Kuberry. Explicit synchronous partitioned algorithms for interface problems based on Lagrange multipliers. Computers & Mathematics with Applications, 2018. In press.