An unfitted finite element solver for a poromechanic problem with a moving boundary

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ABSTRACT

Reconstructing the stress and deformation history of a sedimentary basin is a challenging and important problem in the geosciences for a variety of applications [1]. The mechanical response of a sedimentary basin is the consequence of complex multi-physics processes involving mechanical, geochemical, thermal, geophysical and geological aspects. Among all the inclusion of erosion effects in the reconstruction of a sedimentary basin history play an key role. A sedimentary basin is usually represented by a deformable porous medium saturated with water. To describe erosion, we make the top surface of the basin evolve according to a given profile. The poroelastic material is described by means of the Biot model while the moving boundary accounts for the erosion of the material. We focus on the numerical approximation of the problem in the framework of the finite element method. The poromechanic problem is solved by means of the fixed stress splitting [2], a sequential procedure where the flow is solved first followed by the solution of the mechanical problem. To avoid re-meshing the computational domain along with the evolution of the boundary, we adopt the cut finite element approach (CutFEM) as described in [3]. More precisely, the physical domain is embedded in a larger computational grid and the evolution of top surface of the basin is described by the zero value of a time dependent level set function. The main issue of this strategy consists of the ill-conditioning of the finite element matrices in presence of cut elements of small size. We show, by means of numerical experiments and theory, that this issue significantly worsens the performance of the numerical solver of the discrete problem. For this reason, we propose a strategy that allows to overcome the ill-conditioned behavior of the discrete problem. The resulting solver is based on the fixed stress approach combined with the ghost penalty stabilization [4] and preconditioning applied to the pressure and displacement sub-problems, respectively. The performance of the new solver are investigated through different numerical experiments based on realistic geometries.

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