Adaptive Geometric Multigrid Method for the Finite Cell Method

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ABSTRACT

The finite cell method (FCM) [1] is a well-established ficticious domain method for solving partial differential equations, and octree-based quadrature on subcells can be employed for an accurate integration on cells cut by the original boundary [1, 2]. The computational effort is shifted from mesh generation to the element-wise quadrature, allowing for a straight-forward numerical simulation on complex domains. However, small fractions of the physical domain on cut cells lead to a detoriation of the condition number of the arising linear system of equations which has to be addressed by efficient solvers [3].

In this contribution, we focus on the latter issue for flow and diffusive PDEs. Similar to the *hp-d* FCM approach [4], we employ a rectangular finite element mesh that is adaptively refined towards the embedded boundary. This is intuitively motivated by the observation that more accuracy is required close to complex boundary geometries and to account for boundary layers in flow simluations. In addition, adaptive mesh refinement (AMR) on the cut elements may alleviate the issue of the small cut volume fraction and thus avoid the condition number deterioration. In our work, the arising hanging nodes resulting from AMR are handled by interpolation constraints and a hierarchy of nested finite element spaces is thereby created. We then employ this hierarchy to implement the adaptive geometric multigrid method [5, 6] for the solution of the arising linear systems of equations. The geometric multigrid method is used as a preconditioner in Krylov methods such as CG or BiCGStab. An advantage of this approach is that both adaptive quadrature and the geometric multigrid hierarchy can be implemented on the same tree-like mesh data structure. We present empirical studies evaluating the approach for the Poisson as well as Stokes equation.

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