Locally Adapting Equations in Discontinuous Galerkin for Compressible Flows

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ABSTRACT

Simplifying equations as permitted by physical phenomena is one of the reasons to employ coupling in a simulation. By solving for simpler equations in parts of the computational domain, the computational effort can be reduced. Deciding which equations to solve where a-priori is not often easily possible, and it usually is necessary to overestimate the parts, where the more complex equation is to be employed.

The discontinuous Galerkin method [1] offers a scheme where elements are relatively loosely coupled via fluxes across interfaces, while the internal representation and calculation within elements is not relevant to neighboring elements. We exploit this locality of the scheme and change the equation to solve locally, based on the solution within the element. To represent the solution inside each element, a polynomial approximation is used in Legendre form. This representation enables fast high-order computations for linear systems.

We utilize this scheme to solve compressible flows, where acoustic waves may be generated. The acoustic wave propagation is a linear problem, however the noise generating vortices are governed by the nonlinear Euler equations of compressible flows. Usually, we are interested in large areas across which the acoustic waves are transported, but the nonlinear vortices are only observed in a small fraction of this large domain [2]. With the adaptive scheme, we can limit the more expensive nonlinear computation to those elements in the domain, where the nonlinearity needs to be considered, while solving the simplified linearized equations everywhere else.

We present the adaptive scheme, discuss some possible indicators to decide whether linearization is allowed in elements, present results from simulations with adaptivity and compare it to fully nonlinear simulations. We assess the impact of indicator and the adaptivity on the computational effort for various polynomial degrees.

REFERENCES

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