About imposing the incompressibility constraint at both the Eulerian and Lagrangian levels in immersed boundary methods

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ABSTRACT

The divergence-conforming immersed boundary (DCIB) method is presented to tackle a long-standing issue of immersed boundary (IB) numerical methods for fluid-structure interaction, namely, the challenge of accurately imposing the incompressibility constraint at the discrete level [1]. IB methods deal with incompressible visco-elastic solids interacting with incompressible viscous fluids. The DCIB method follows up on our previous works [2-3], where we developed discretizations of the mathematical model proposed by the IB method based on non-uniform rational B-splines (NURBS) and T-splines, respectively. In the DCIB method, the Eulerian velocity-pressure pair is discretized using divergence-conforming B-splines, which leads to *inf-sup* stable, H^1-conforming, and *pointwise* divergence-free Eulerian solutions. The Lagrangian displacement is discretized using NURBS, which enables to robustly handle large mesh distortions. The data transfer needed between Eulerian and Lagrangian descriptions is performed at the quadrature level by using the same spline basis functions that define the computational meshes, conducing to a fully variational formulation, sharper treatment of the fluid-solid interface, and a 0.5 increase in the convergence rate of the Eulerian velocity and the Lagrangian displacement measured in L^2 norm. By combining the generalized-α method and a block-iterative solution strategy, the DCIB method results in a fully-implicit discretization, which is key to accurately impose the incompressibility constraint at the Lagrangian level.

Various two- and three-dimensional problems are solved to show all the above-mentioned properties of the DCIB method together with mesh-independence studies, comparisons with other methods from the literature, and measurement of convergence rates. The DCIB method leads to completely negligible incompressibility errors at the Eulerian level and various orders of magnitude of increased accuracy at the Lagrangian level compared to other IB methods.

REFERENCES

[1] H. Casquero, Y. J. Zhang, C. Bona-Casas, L. Dalcin, H. Gomez, Non-body-fitted fluid-structure interaction: Divergence-conforming B-splines, fully-implicit dynamics, and variational formulation, *Journal of Computational Physics*, **374**, 625-653, 2018.

[2] H. Casquero, C. Bona-Casas, H. Gomez, A NURBS-based immersed methodology for fluid-structure interaction. *Computer Methods in Applied Mechanics and Engineering*, **284**, 943-970, 2015.

[3] H. Casquero, L. Liu, C. Bona-Casas, Y. J. Zhang, H. Gomez, A hybrid variational-collocation immersed method for fluid-structure interaction using unstructured T-splines. *International Journal for Numerical Methods and Engineering*, **105**, 855-888, 2016.