A robust numerical method for multi-physics initial boundary value problems

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ABSTRACT

Energy-stable numerical methods for initial boundary value problems (IBVP) often rely on a summation-by-parts (SBP) property. The SBP property ensures that the discrete differential operators mimic the integration-by-parts formula for their continuous counterparts. Examples of discrete operators with the SBP property include some finite difference (FD)[1], spectral collocation [2], and finite volume methods [3]. SBP FD operators are routinely combined with the simultaneous approximation term (SAT) method to impose boundary conditions. The combination of SBP operators and the SAT technique allow us to construct numerical methods that exactly mimic the energy growth of the underlying IBVP. The SBP-SAT method is a robust numerical technique for multi-physics problems and hybrid numerical methods, combining different SBP discretisations. Another method of imposing boundary conditions in the framework of SBP, that has recently gained attention is the the projection method [4].

In [5] a multi-physics model is constructed for simulation of acoustic and flexural gravity waves. Such waves arise when studying ocean wave interactions with floating ice shelves, sea ice, and floating structures. Particular emphasis is on a well-posed interface treatment of the fluid-ice coupling. To ensure numerical stability and efficiency, a high-order accurate SBP-SAT method is employed to impose the boundary and the multi-physics interface conditions.

In the present study we will show how the SBP-SAT (or Projection) method can be employed to ensure numerical stability and efficiency in the framework of well-posed multi-physics IBVP. As a proof of concept we will show results from the analysis and convergence study of acoustic and flexural gravity waves in compressible, inviscid fluids partially covered by a thin elastic layer [5]. We will also show some recent results concerning a coupled (multi-physics) model of a seismic air gun, solved with the SBP-SAT method. This is an extension of the work presented in [6].

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