Partitioned multirate domain decomposition waveform relaxation methods for the heat equation

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ABSTRACT

The efficient simulation of thermal interaction between fluids and structures is crucial in the design of many industrial products, e.g. thermal anti-icing systems of airplanes, gas quenching, which is an industrial heat treatment of metal workpieces, or the cooling of rocket thrust chambers.

We model unsteady thermal fluid structure interaction using two partial differential equations, one of them describes the fluid and the other one the structure. Coupling conditions are introduced at the interface to connect fluid and structure. These coupled problems can be simulated by the partitioned approach that uses existing codes for each of the fields and coordinates the solution across the interface in an iterative manner. The library preCICE [1] that supports the simulation of surface-coupled multi-physics phenomena through a partitioned black-box approach. However, being able to use different time step sizes for each of the fields will allow a more efficient partitioned simulation.

We compare here two domain decomposition waveform relaxation based approaches to build a high order multirate time stepping scheme. The first approach is based on the Dirichlet-Neumann coupling [4] while the other is based on the Neumann-Neumann coupling [3]. The partitioned heat equation with jumping coefficients is used as a model problem. Both schemes are implemented using preCICE.

In particular, we compare the high order multirate Dirichlet-Neumann (DNWR) and Neumann-Neumann (NNWR) waveform relaxation methods for two heterogeneous coupled heat equations. Both algorithms were introduced by [2] for the nonmultirate case and the optimal relaxation parameter was determined for the coupling of homogeneous materials. In the case of heterogeneous materials, we present here analyses of the DNWR and NNWR algorithms showing that the optimal relaxation parameter is highly dependent on the material coefficients [3].

REFERENCES

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