Dynamic Mode Decomposition for the Monodomain Sub-Model in the Neuro-Muscular System

N. Emamy*,[1], M. Widmayer[1], B. Maier[1], T. Klotz[2], O. Röhrle[2] and M. Mehl[1]

[1] Institute for Parallel and Distributed Systems, University of Stuttgart
Universitätsstr. 38, 70569 Stuttgart, Germany
e-mail: nehzat.emamy@ipvs.uni-stuttgart.de, miriam.mehl@ipvs.uni-stuttgart.de
Web page: https://www.ipvs.uni-stuttgart.de/abteilungen/sgs/

[2] Institute of Applied Mechanics (CE), SimTech Research Group on Continuum Biomechanics and Mechanobiology, University of Stuttgart
Pfaffenwaldring 5a, 70569 Stuttgart, Germany
e-mail: thomas.klotz@mechbau.uni-stuttgart.de, roehrle@simtech.uni-stuttgart.de
Web page: http://www.mechbau.uni-stuttgart.de/ls2/jrg/index.html

ABSTRACT

The neuro-muscular system is a complex multiscale coupled system, for which detailed biophysics-based chemo-electro-mechanical models are available. Realistic simulations of such models are computationally extremely demanding, see, e.g., [1] and references therein. The 3D continuum-mechanical problem, which describes the deformation of the muscle, is coupled with a 1D problem, which describes the propagation of the action potential along muscle fibers. The propagation of action potentials can be described with the monodomain model, consisting of a partial differential equation (PDE) and associated ordinary differential equations (ODEs), representing the ionic currents across the muscle cell membranes. The ODEs derived from a Hodgkin-Huxley type [2, 3] modeling approach are stiff and thus require very small time steps.

To accelerate the simulations, we have recently applied a POD-DEIM model reduction technique to the above mentioned problem [4]. Considering a total reduction of the action potential, together with the ODE state variables, there is a high potential for reducing the coupled system. However, the system is solved for large physical time intervals with repeated stimulation of the muscle fibers. This motivates extrapolation of the solution of the monodomain equation in time instead of solving it in each activation period. For this purpose, the higher order dynamic mode decomposion (HODMD) [5, 6] seems to be an appropriate method, which uses time-lagged snapshots. A tunable parameter $d \geq 1$ (number of related time-lagged snapshots) has to be adopted for accuracy. With $d = 1$ one obtains the classical dynamic mode decomposition (DMD). To the best of our knowledge, the DMD/HODMD is not yet studied in context of neuro-muscular problems. We consider a 1D muscle fiber and stimulate it in the middle with different frequencies. The simulations are performed for different spatial and temporal resolutions. We use these full-order solutions and consider several values of d to find out an appropriate basis for the reduced solution. The first results show that the HODMD is more accurate than DMD in capturing the underlying physical modes of solution and basically applicable to neuro-muscular problems.

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