A method to solve wave equations with multisingular vortex Gaussian initial conditions

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ABSTRACT

We present a method to solve a wave equation in two transversal dimensions plus one evolution variable, when the initial condition is a multisingular vortex Gaussian beam. This initial condition is a complex field carrying a number of phase singularities in a plane transversal to propagation embedded in a Gaussian beam. This method is relevant to, e.g., quantum mechanics and optics. The wave equation appears in quantum mechanics, and then is named the Schrödinger equation; in optics it is the paraxial wave equation, also called often called the Fock-Leontovich equation [1, 2]. The initial condition is also relevant to both fields. The first step is to construct the scattering modes that represent the initial condition [3]. To this end, one needs to obtain a particular set of polynomials, which play an analogous role to Laguerre polynomials for Laguerre–Gaussian modes. These polynomials can be constructed straightforwardly using the recurrence relations introduced in [3]. These scattering modes are then propagated to any value of the evolution variable allowing to obtain the solution during all propagation. More importantly, they allow to obtain equations for the positions of the phase singularities. This allows for the derivation of the trajectories followed by the singularities, also called dark rays in optics [5, 4]. These dark rays explain experimental effects such as vortex transmutation, that occurs in photonic crystal fibers with abrupt changes in the symmetry of the fiber [5, 4, 6]. We illustrate the method with several examples. We also give a numerical comparison between the results obtained from the method and the propagation of such an initial beam by a nonlinear Schrödinger equation. The Non-linear Schrödinger equation appears in, e.g., nonlinear optics and in ultracold atoms, where it is called Gross-Pitaevskii equation. In both fields, multisingular initial conditions such as those treated here are experimentally relevant [7]. While the method discussed here describes correctly the dynamics at short times, it fails to describe full dynamics.

REFERENCES

[1] R. Fedele and M. A. Man'ko, Beam optics applications: quantumlike versus classical-like domains, *Eur. Phys. J. D* (2003) **27**:263–271.

[2] M. A. Man'ko, Analogs of time-dependent quantum phenomena in optical fibers, *J. Phys.: Conf. Ser.* (2008) **99**: 012012.

[3] A. Ferrando and M.A. Garcia-March *J. Opt.* (2016) **18**:064006.

[4] A. Ferrando A Phys. Rev. A (2014) **90**:023844.

[5] A. Ferrando and M.A. Garcia-March *J. Opt.* (2013) **15**:044014.

[6] D. Novoa, I.J. Sola, M.A. Garcia-March, and A. Ferrando *Appl. Phys. B* (2014) 116:779.

[7] K. A. Commeford, M. A. Garcia-March, A. Ferrando, and Lincoln D. Carr, *Phys. Rev. A* (2012) **86**: 023627.