Embedded multilevel Monte Carlo for uncertainty quantification in complex random domains

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ABSTRACT

Uncertainty quantification requires the solution of a stochastic partial differential equation (PDE) with random data, e.g. material properties (PDE parameters), boundary and/or initial conditions and the geometry of the domain. In real situations, geometry is hardly known exactly and surfaces are rarely smooth when looking at sufficiently small scale. This uncertainty can have a strong impact on the solution of a PDE.

There are several methods for solving stochastic PDEs such as stochastic Galerkin or stochastic collocation but most of them suffer the so-called "curse of dimensionality", a dramatic increase of computational cost with the number of stochastic variables. Besides, some of these methods are intrusive: a code that can be used to solve a deterministic problem needs to be modified to solve a stochastic one. In contrast, the classical Monte Carlo method does not present these drawbacks but statistical and spatial accuracy requirements make it prohibitively expensive.

To address this difficulty the multilevel Monte Carlo (MLMC) method [3] exploits a hierarchy of meshes to compute expectations on each level with a decreasing number of samples (in the finer meshes), thus reducing the computational cost by orders of magnitude. It has been successfully applied to stochastic PDEs with random material properties and/or initial and boundary data.

There are three approaches to deal with geometric uncertainty. The first one is based on domain mappings to transform a problem in a random domain into a stochastic problem in a deterministic domain. Since it requires the construction of a global smooth mapping, its application is restricted to simple geometries. The second one is based on introducing perturbations (e.g. a Taylor expansion using shape derivatives) to a nominal domain. As any perturbation method, some restrictions on the perturbation size need to be assumed, reducing its applicability.

In this work we follow the third approach, exploiting the generality of embedded methods, as proposed in [2]. We discuss how to exploit unfitted finite element techniques in the context of the MLMC method to perform uncertainty quantification when complex geometries with random boundaries are considered. The traditional ill-posedness of these techniques is cured using aggregation techniques for conforming methods [1].

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