

# On the modeling impacts of the energy equation in the simulation of melting

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## ABSTRACT

Despite tremendous works devoted to the modeling and the simulation of solid-liquid phase change, it appears that many questions are still unsolved or under debate. Although some physical phenomenon are common in both transitions, it is usual to differentiate melting from solidification to address such problems.

Thus, in the present study, we will be focused only in fusion, and more specifically, to its macroscopic modeling and simulation with one-fluid methods, that is to say in a fixed grid context.

The starting point will be a discussion about the various ways to model the energy equation, namely the enthalpy formulations [1, 2], the source-based formulations [3, 4, 5], the apparent capacity formulations [6, 7, 8] and the complete enthalpy formulations [9, 10, 11]. Based on a previous study, we will present an exhaustive benchmark of these latter closures on both analytical solutions (one-phase and two-phase Stefan problems) and experimental measurements. The aims are here manifold: i) to propose the first quantitative comparison between all these models, ii) to develop some confidence or to better know the possible weaknesses associated to these models, iii) to propose some quality thresholds to help engineers using commercial softwares (implementing one of these models) when simulating such situations. Influence of the thermodynamical modeling is also tackled, and especially the role of the function retained for the equation of state together with the associated parameters.

To extent the purpose, we will add some new results based on the modeling of the convective term, which is clearly different with fixed grid methods, *i.e.* diffuse methods, than it could be with other methods (front tracking, phase field...).

Eventually, let us mention that these various topics will also be discussed from the computational point of view. Then, total time of calculations, number of iterations and influence of the various way to settle the convergence will be analyzed.

Last but not least, we will conclude on the difficulty to validate thoroughly the numerical results and the need for such a protocol in current simulations.

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