A variational framework for thermo-mechanical coupled two-scale boundary value problems

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ABSTRACT

We propose a variational framework for thermo-mechanical coupled two-scale boundary value problems, which enables us to analyze both mechanical and thermal behaviors for composite materials simultaneously. This framework can be regarded as an extended version of the thermo-mechanical coupled incremental variational framework [1] and is constructed by applying the so-called generalized Hill-Mandel principle [2], which let us consider microscopic inertia effects. Specifically, we construct our proposed framework by beginning with introducing the Hill-Mandel principle which the macroscopic thermo-mechanical coupled rate potential is equivalent to the volume average of the microscopic one. After, in line with [2], we formulate corresponding two-scale Euler-Lagrange equations by explicitly employing it with the appropriate boundary conditions for the microstructure.

The primary benefit of our proposed framework is that all of the properties of a variational structure are inherited. For instance, the symmetry of Hessian in the global problem, i.e., the two-scale consistent tangents, is guaranteed, and thus, in the case of employing the Newton-Raphson method, it offers us numerical efficiency. Furthermore, if selected materials are "variational" in the sense of [1], the heat conversion efficiency from the inelastic dissipation is automatically calculated without additional formulations.

The accuracy of our proposed framework is validated by solving several numerical examples for thermo-hyperelastic and standard dissipative composites. In particular, we show its advantages by comparing with the conventional approach [3], which is close to our proposed framework.

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