The Brezis-Ekeland-Nayroles minimization principle with mixed finite element method for elastoplastic dynamic problems

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* Laboratoire de Mécanique, Multiphysique, Multiéchelle (LaMcube) FRE CNRS 2016
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ABSTRACT

Most computer-aided engineering software provide a classical incremental computation procedure for nonlinear problems. However, computation error accumulates during the process which can make the simulation fail sometimes. Although little used in the literature, the Brezis-Ekeland-Nayroles (BEN) [1,2,3] principle, an alternative step-by-step algorithm, based on the time integration of the sum of the dissipation potential and its Fenchel polar can have a global view of whole evolution. In short, the BEN principle converts a mechanical problem to a constrained optimisation problem. Recently, Buliga and de Saxcé [4] have proposed a symplectic version of the BEN principle which generalizes the Hamiltonian inclusion formalism for the dissipative systems.

In the present work, this formalism is specialized to the standard plasticity in small, finite strains, in statics and dynamics. We apply it to solve the classical problem of a thick tube in plane strain subjected to an internal pressure. The tube is discretized with three mixed finite elements, stress, displacement and plastic multiplier rate fields.

The choice of a polynomial statically admissible stress field is guided by the aim to avoid the global (or structural) equilibrium equations in the constrained minimization problem. Only remains the local yield condition. The displacement field is chosen to provide a strain field with the same number of parameters as the one of the stress field. The plastic deformation rate provided by plastic multiplier rate is employed on the additive decomposition of strain rates.

An excellent agreement is obtained between the numerical results obtained by the BEN approach and the analytical solution for a thin tube.

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