

# Gradient Crystal Plasticity Models with a Natural Length Scale at the Slip-Rate Gradient in the Hardening Law

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## ABSTRACT

In the present work, a class of crystal plasticity models is examined which involve the second-order gradient of plastic slip or slip rate in the condition for plastic flow, cf. [1]. The approach requires the use of an energetic or dissipative internal length scale, usually assumed arbitrarily. As an extension of this formulation, the first-order gradient of plastic slip rate is incorporated here in the incremental form of the hardening law. This is associated with another, natural length scale that is not arbitrary but has recently been derived in a closed form from phenomenological relationships of the dislocation theory of plasticity [2]. Both approaches are combined in a thermodynamically consistent way by exploiting compatibility of the actual and virtual dissipation rates.

The numerical implementation of the resulting models is based on the dual-mixed formulation [3], in which auxiliary global variables, that are counterparts to slip gradients, are introduced as additional degrees of freedom. The displacement field and the fields of these auxiliary variables are discretized by using a linear FE-approximation. The equation of the balance of momentum coupled with the additional equations for auxiliary variables are solved within a monolithic iterative solution strategy using the Newton method. The applied numerical technique is verified by showing the agreement of 1D and 2D results, obtained in the absence of the natural length scale, with the earlier results presented in [1] and [4], respectively.

The interplay between the length-scales incorporated in the model is examined and illustrated by the examples of monotonic and cyclic deformation of one- and two-dimensional models of Cu single crystals with boundary constraints imposed on plastic slips. It has been shown [5] that both length scales can have significant and different influence on the stress-strain response as well as on the distribution of plastic slips and geometrically necessary dislocation (GND) densities within the body.

## REFERENCES

- [1] M.E. Gurtin, L. Anand and S.P. Lele “Gradient single-crystal plasticity with free energy dependent on dislocation densities”, *J. Mech. Phys. Solids*, **55**, 1853-1878 (2007).
- [2] H. Petryk and S. Stupkiewicz “A minimal gradient-enhancement of the classical continuum theory of crystal plasticity. Part I: The hardening law”, *Arch. Mech.*, **68**, 459-485 (2016).
- [3] M. Ekh, M. Grymer, K. Runesson and T. Svedberg “Gradient crystal plasticity as part of the computational modelling of polycrystals”. *Int. J. Numer. Meth. Eng.*, **72**, 197–220 (2007).
- [4] S. Bargmann, B.D. Reddy and B. Klusemann “A computational study of a model of single-crystal strain-gradient viscoplasticity with an interactive hardening relation”, *Int. J. Solid Struct.*, **51**, 2754-2764 (2014).
- [5] M. Rys and H. Petryk “Gradient crystal plasticity models with a natural length scale in the hardening law”, *Int. J. Plast.*, **111**, 168-187 (2018).