Elastoplasticity with linear tetrahedral elements:  
A variational multiscale method

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ABSTRACT

We present a computational framework for the simulation of J2-elastic/plastic materials in complex geometries based on simple piecewise linear finite elements on tetrahedral grids [1]. We avoid spurious numerical instabilities by means of a specific stabilization method of the variational multiscale kind. Specifically, we introduce the concept of subgrid-scale displacements, velocities, and pressures, approximated as functions of the governing equation residuals. The subgrid-scale displacements/velocities are scaled using an effective (tangent) elastoplastic shear modulus, and we demonstrate the beneficial effects of introducing a subgrid-scale pressure in the plastic regime. We provide proofs of stability and convergence of the proposed algorithms. These methods are initially presented in the context of static computations and then extended to the case of dynamics, where we demonstrate that, in general, naive extensions of stabilized methods developed initially for static computations seem not effective. We conclude by proposing a dynamic version of the stabilizing mechanisms, which obviates this problematic issue. In its final form, the proposed approach is simple and efficient, as it requires only minimal additional computational and storage cost with respect to a standard finite element relying on a piecewise linear approximation of the displacement field.

REFERENCES