Preconditioning and error estimation for spectral homogenization solvers

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ABSTRACT

Since the seminal work of Moulinec and Suquet [1], spectral solvers have become a popular choice in the computational homogenization of heterogeneous media. In this talk, we focus on variationally-based Newton-type methods [2, 3] based on the linearization of non-linear nodal balance equations resulting from a trigonometric Galerkin discretization. The bulk of the computational effort thus concentrates in the iterative solution of the ensuing system of linear algebraic equations using, e.g., the Conjugate Gradient (CG) method considered in what follows.

We discuss two options for reducing the number of iterations. The first one builds on exploring the ideas of (generalized) Laplace preconditioning of elliptic operators in the context of spectral homogenization solvers, and the notion of the reference problem [1], in particular. The second option relies on balancing discretization and algebraic errors, resulting from the Galerkin discretization and an approximate solution to the linear system, respectively. We demonstrate with simple examples that both options allow us to significantly reduce both the number of CG iterations and the solution time while requiring only minor changes to existing solvers.

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