Fractional Strain-Gradient Plasticity

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ABSTRACT

Prompted by the mounting experimental evidence pointing to a gaping discrepancy between the size-dependent yield strength of metals and predictions from conventional strain-gradient plasticity (SGP), e. g. [1, 2, 3], we have developed [4] a new *fractional strain-gradient theory of plasticity* (FSGP) that uses fractional derivatives of plastic strain as a means of quantifying the inhomogeneity of plastic deformation.

A common form of the size dependent yield stress can be written

$$\sigma_{\mathrm{y}} = \sigma_0 \left[1 + \left(\frac{\ell}{h} \right)^{\alpha} \right], \tag{1}$$

where ℓ is a material length scale, h is an appropriate size measure of the plastically deforming region and α is a scaling exponent. Conventional SGP almost invariably predicts a size scaling exponent $\alpha = 1$ that, in many cases, grossly overestimates the experimentally-observed values. We take this discrepancy to suggest that the differential structure of conventional strain-gradient plasticity is overly stiff and proceed to relax the excessive rigidity by recourse to fractional plastic-strain gradients. Specifically, by allowing the free energy to depend on a fractional derivatives of strain, we show that the size-scaling discrepancy between conventional SGP and the experimental data in [2, 3] is resolved. When applied in the shear layer configuration, the theory predicts a size scaling relation with exponent α equal to the fractional order of plastic strain-gradient differentiation. Through this identification, the observed experimental scaling can be exactly matched by an appropriate choice of fractional differential order.

The form of the non-local fractional plastic strain-gradient contribution to the free energy is explicitly given by a double-integral representation and its fractional differential character is set simply by an appropriate choice of exponents in the interaction kernel.

REFERENCES

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