Construction of general multiscale methods using dimensionality reduction and domain decomposition

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This work presents a method for constructing general multiscale methods for periodic structures. Contrary to classical approaches, such as first order homogenization, its scope is independent of both the size of the unit cell and the external forces acting on the structure. The flavor of the approach is to regard the equilibrium of the structure as a domain decomposition problem in which simplifications are made in the way subdomains deform, and also in the way subdomains interact with each other. The employed domain decomposition technique is the so-called *Localized Lambda Method* [2, 3]. Domain displacements, Lagrange multipliers and interface displacements are approximated as linear combinations of *dominant* modes. Such modes, in turn, are obtained via dimensionality reduction tools (the Singular Value Decomposition) from 3D numerical experiments. Evaluation of the tangent stiffness matrix of each domain is made using a reduced-order integration technique —the *Empirical Cubature Method* [1]. It is shown that the computational cost of solving the partitioned system depends only on the number of domains in the partition, but not on the size of the finite element discretization of each partition. Recovery of 3D fields can be made in a post-process stage via simple matrix multiplication. The performance of the method is illustrated with prismatic and cellular structures. It is shown that in the former case (in which the subdomains are "slices"), if the fictitious interface are constrained to remain plane, the resulting system of equations has the same format as the one obtained in a classical finite element beam method, yet with the remarkable difference that the proposed scheme furnishes stress and displacement fields that are totally *consistent* with results computed using a full-order, 3D formulation.

REFERENCES

[1] JA Hernández, MA Caicedo, and A Ferrer. Dimensional hyper-reduction of nonlinear finite element models via empirical cubature. *Computer Methods in Applied Mechanics and Engineering*, 313:687–722, 2017.

[2] KC Park and Carlos A Felippa. A variational framework for solution method developments in structural mechanics. *Journal of Applied Mechanics*, 65(1):242–249, 1998.

[3] KC Park and Carlos A Felippa. A variational principle for the formulation of partitioned structural systems. *International Journal for Numerical Methods in Engineering*, 47(1-3):395–418, 2000.