A mixed energetic/dissipative higher-order potential for strain-gradient plasticity under non-proportional loading

Andrea Panteghini*, Lorenzo Bardella^ and Christian F. Niordson†

*,^ Department of Civil, Environmental, Architectural Engineering and Mathematics (DICATAM)
University of Brescia
Via Branze 43, 25123 Brescia, Italy
e-mail: andrea.panteghini@ing .unibs.it, lorenzo.bardella@ing.unibs.it

† Department of Mechanical Engineering, Solid Mechanics
Technical University of Denmark
DK-2800 Kgs. Lyngby, Denmark
e-mail: cn@mek.dtu.dk

ABSTRACT

Higher-order strain gradient plasticity (SGP) theories allow for modelling experimentally observed size effects in small-scale metal plasticity. Their main features include the capability to predict the increased hardening and yield stress (strenghtening) characterizing the plastic response of metals at small scales in the presence of plastic strain gradients (see for instance [1] and references therein). These SGP theories employ higher-order boundary conditions that may be used to impose constraints on plastic deformation due to dislocations blockage leading to pileups.

Since the pivotal contribution of Fleck et al. [2], it has been reported that the so-called *non-incremental* SGP theories may suffer a so called *elastic gap* due to the specific way in which higher-order dissipative stresses enter these theories (see, e.g., [3, 4]). The *elastic gap* is a purely elastic incremental response occurring in connection with an abrupt change in load path (characterizing the type of non-proportional loading). The elastic gaps so far documented in the literature occur after sudden variation in either conventional loading path, or higher order boundary conditions.

It has been found that dissipative higher-order stresses lead to strengthening as observed on small scale experiments (see [5] and references therein). Regarding energetic higher-order stresses, strengthening is only observed if one resorts to a less-than-quadratic defect energy governing the (recoverable) higher-order stresses [4,6,7]. However, in this case, SGP theories predict anomalous cyclic behavior with inflection points where strain gradients are small [4,8].

In this work we first show that in cyclic plasticity an elastic gap may occur also by negleting higher-order dissipative stresses if a less-than-quadratic defect energy is employed.

As a main contribution, in order to overcome the foregoing limits, we propose a new thermodynamically consistent higher-order plastic potential that allows us to predict reliable cyclic response and energetic strenghtening without exhibiting elastic gaps. The new potential includes both energetic and dissipative contributions. The features of the new formulation, that can be employed in any higher-order SGP theory, are here demonstrated and discussed by referring to the Gurtin 2004 SGP theory [9,10], which is based on the use of Nye's dislocation density tensor as primal higher-order kinematic variable, thus constitutively involving the plastic spin.

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