High-Performance Interior Point Methods for Three-Dimensional Finite Element Limit Analysis

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ABSTRACT

The ability to obtain rigorous upper and lower bounds on collapse loads of various structures makes finite element limit analysis an attractive design tool. The increasingly high cost of computing those bounds, however, has limited its application on problems in three dimensions. This work reports on a high-performance homogeneous self-dual primal-dual interior point method developed for three-dimensional finite element limit analysis. This implementation achieves convergence times over 4.5× faster than the leading commercial solver across a set of three-dimensional finite element limit analysis test problems, making investigation of three dimensional limit loads viable. A comparison between a range of iterative linear solvers and direct methods used to determine the search direction is also provided, demonstrating the superiority of direct methods for this application.

The components of the interior point solver considered include the elimination of and options for handling remaining free variables, multifrontal and supernodal Cholesky comparison for computing the search direction, differences between approximate minimum degree [1] and nested dissection [2] orderings, dealing with dense columns and fixed variables, and accelerating the linear system solver through parallelisation. Each of these areas resulted in an improvement on at least one of the problems in the test set, with many achieving gains across the whole set. The serial implementation achieved runtime performance 1.7× faster than the commercial solver Mosek [3]. Compared with the parallel version of Mosek, the use of parallel BLAS routines in the supernodal solver saw a 1.9× speedup, and with a modified version of the GPU-enabled CHOLMOD [4] and a single NVIDIA Tesla K20c this speedup increased to 4.65×.

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