# Application of mixed least-squares finite element formulations for small and finite strain elasto-plasticity

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## ABSTRACT

In this contribution mixed least-squares finite element formuluations for small and finite strain elasto-plasticity are presented. The stress approximation plays a crucial role in the framework of plasticity since it is mainly responsible for the evolution of plastic material response. In the proposed mixed approach the stresses and the displacements are introduced as unknown fields using the least-squares method. The least-squares method provides some numerical advantages, among others the flexibility in the construction of functionals which are directly approximating suited unknown field variables. Furthermore, it is given as a minimization problem and thus not restricted by the LBB-condition which yields the possibility to choose the interpolation order and combinations without any constraints. The method is based on a $L^2$-norm minimization of the residuals of a first-order system of differential equations, see e.g. [1]. The considered residuals are the balance of momentum, an elasto-plastic constitutive relation and a mathematically redundant residual enforcing the stress symmetry which is in general not fulfilled a priori due to the chosen stress approximation by Raviart-Thomas functions. The consideration of the least-squares formulation based on elasto-plasticity could yield to discontinuities within the first variation based on the plastic constraints in the constitutive relation, see e.g. [2]. A possible approach to overcome this drawback is a modification of the first variation leading to a continuous modified weak form. For the implementation of the theory of plasticity we choose for simplicity an isotropic von Mises yield criterion with linear hardening. The derivation of the flow rule for finite strain is based on an implicit exponential time integration scheme for the inverse plastic right Cauchy-Green deformation tensor, see [3]. The hardening law as well as the Kuhn-Tucker conditions are treated in the same way for the small and finite strain model, see e.g. [4].

## REFERENCES

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