Thermo-Mechanically Coupled and Uncoupled Simulations using High-Order Time-Integration Schemes

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ABSTRACT

Constitutive equations of rate-type (rate-independent plasticity, viscoelasticity and viscoplasticity) combined with the weak form of the equilibrium conditions in the sense of finite elements yield a system of differential-algebraic equations after the spatial discretization. Commonly these are solved using a Backward-Euler method on Gauss-point level to integrate the evolution equations and combining it with the Multilevel-Newton method (MLNA), see [1]. This is extended in [1] by error-controlled, time-adaptive finite element simulations using higher order diagonally-implicit Runge-Kutta methods (DIRK), which are applied to the entire DAE-system since in each stage a Backward-Euler-like structure results.

The coupling of the equilibrium conditions with the transient heat equation leads to a DAE-system again. Thus, the same high-order schemes can be applied, which is discussed in [2]. However, problems with non-linear Dirichlet boundary conditions might occur due to an order reduction phenomenon. This, however, can be circumvented by integrating the boundary conditions. The partitioned approach, where staggered schemes are applied, is embedded in the DIRK/MLNA schemes as well.

The combination of the transient heat equations with constitutive models of thermo-viscoplasticity under the assumption of dynamical situations – the inertia terms are considered now as well – yield a coupled system of second and first order ordinary differential equations. Here, both a DIRK/MLNA and a generalized-α/MLNA approach are possible to be applied, see [3]. Classical approaches using Newmark-schemes combined with Backward-Euler schemes on Gauss-point level do not reach second order. We show error-controlled, time-adaptive computations reaching the expected order in all variables (displacements, velocities, temperatures, and internal variables). Thus, step-size control is available in dynamics of thermos-mechanical coupled plasticity problems as well.

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