

# **A Quasi-inextensible and Quasi-incompressible Finite Element Formulation for Transversely Anisotropic Hyperelastic Solids and Soft Biological Tissues**

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## **ABSTRACT**

The contribution presents a novel finite element formulation for quasi-inextensible and quasi-incompressible finite hyperelastic behaviour of transversely anisotropic materials and addresses its computational aspects. The formulation is presented in purely Eulerian setting and based on the additive decomposition of the free energy function into isotropic and anisotropic parts where the former is further decomposed into isochoric and volumetric parts. For the quasi-incompressible response the Q1P0 element formulation is outlined briefly where the pressure-type Lagrange multiplier and its conjugate enter the variational formulation as an extended set of variables. Using the similar argumentation an extended Hu-Washizu-type mixed variational potential is introduced where the volume averaged fiber stretch and fiber stress are additional field variables. Within this context, the resulting Euler-Lagrange equations and the element formulation resulting from the extended variational principle are derived [1]. The numerical implementation exploits the underlying variational structure leading to a canonical symmetric structure. The efficiency of the proposed approach is demonstrated through representative boundary value problems. The superiority of the proposed element formulation over the standard Q1- and Q1P0-element formulation is studied through convergence analyses. The proposed finite element formulation is modular and shows excellent performance for fiber-reinforced elastomers in the inextensibility limit. We demonstrate the performance of the proposed formulation in terms of representative boundary value problems applied to (i) fiber-reinforced elastomeric solids and (ii) soft biological tissues.

## **REFERENCES**

- [1] H. Dal, *Quasi-incompressible and quasi-inextensible element formulation for transversely anisotropic materials*, *Int. J. Num. Meth. Engng*, under review.