

The Schwarz Alternating Method in Solid Mechanics

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We advance the Schwarz alternating method as a means for concurrent multiscale coupling in finite deformation solid mechanics. We prove that the Schwarz alternating method converges to the solution of the problem on the entire domain and that the convergence rate is geometric provided that each of the subdomain problems is well-posed, i.e. their corresponding energy density functions are quasi-convex.

It is shown that the use of a Newton-type method for the solution of the resultant non-linear system leads to two kinds of block linearized systems, depending on the treatment of the Dirichlet boundary conditions. The first kind is a symmetric block-diagonal linear system in which each diagonal block is the tangent stiffness of each subdomain, i.e. the off-diagonal blocks are all zero and the coupling terms appear only on the right-hand side. The second kind is a nonsymmetric block system with off-diagonal coupling terms.

Several variants of the Schwarz alternating method are proposed for the first kind of linear system, including one in which the Schwarz alternating iterations and the Newton iterations are combined into a single scheme. This version of the method is particularly attractive, as it lends itself to a minimally intrusive implementation into existing finite element codes.

Finally, we demonstrate the performance of the proposed variants of the Schwarz alternating method on several one-dimensional and three-dimensional examples.

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