

# Plastic length scale evolution

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## ABSTRACT

Here an extension to an isotropic strain gradient plasticity (SGP) theory [1,2] is presented with the aim of incorporating a hypothesis regarding the length scale,  $\ell$ , that usually appears as a constant phenomenological constitutive parameter. Theoretical analysis [3] and experimental evidence [4] of the relation between the geometrically necessary dislocation (GND) density,  $\rho_g$ , and the microstructure length scale,  $\Lambda$ , of dislocation patterning indicates that  $\Lambda \propto \rho_g^{-1/2}$ . Under the assumption that the constitutive length scale  $\ell$  is closely related to the microstructural length scale  $\Lambda$ , up to an multiplicative constant, one can write

$$\ell = A \frac{1}{\sqrt{\rho_g}}$$

where  $A$  is a dimensionless constant. It is now obvious that the length scale is not a constant but will evolve with accumulation of dislocations as

$$d\ell = -\frac{\ell^3}{2A^2} d\rho_g.$$

In the context of isotropic SGP it is reasonable to make a connection between gradients of plastic strain (denoted by  $\eta^p$ ) and the dislocation density  $\rho_g$ . An increment of plastic strain gradient can therefore be related to a change of the GND density, i.e.  $d\rho_g = f(d\eta^p)$ . Furthermore, an extension to include the effects of statistically stored dislocations will be discussed briefly.

Three different functional forms of  $f(d\eta^p)$ , two linear and one quadratic, will be presented. The different models have been implemented in a plane strain FEM code and numerical solutions for pure bending, a biaxially loaded cylindrical void and a cracked plate under uniaxial straining will be presented. The focus will be on how the plastic strain field evolves, specifically with respect to possible plastic localization phenomena, and the implications on the macroscopic stress-strain curve.

## REFERENCES

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