Efficient bridging of the scales in computational solid mechanics: Variationally Consistent Homogenization and Numerical Model Reduction

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ABSTRACT

Numerous problems in engineering and science involve partial differential equations that need to be resolved on vastly separated length and time scales. The use of computational homogenization allows for accounting for the fine-scale features on a micro-scale, while satisfying the pertinent balance equations on the macro-scale, without the need for resolving all features in one single computational domain.

In this contribution, we will adopt the technique of Variationally Consistent Homogenization (VCH), cf. Larsson et al. [1], as a quite general means of constructing schemes for computational homogenization. By introducing the kinematics of chosen homogenized fields on the macro-scale, the effective macro- and microscale problems can be derived from the weak form of the original problem. Furthermore, if the the original problem is derived from a potential, VCH allows for the identification of a generalized macro-homogeneity condition, whereby the variational structure of the problem can be retained in the two-scale formulation.

For linear stationary problems, computational homogenization can be conducted in an "off-line" phase, where by the effective coefficients for the macroscale problem can be pre-computed. However, for the case of a non-linear and/or transient problem, the macroscale and microscale problems have to be solved for concurrently. Although significantly more efficient than resolving the original problem one the finest length-scale, this results in extremely demanding computations.

In order to further decrease the computational cost of the problem, we introduce Numerical Model Reduction on the (discrete) micro-scale problems by adopting a reduced basis approximation. In order to assess the introduced approximation, goal-oriented error estimators are implemented for the problem. For the special case of linear transient problems, guaranteed bounds for the error due to the NMR approximation has been developed, cf. Ekre et ak. [2].

A few applications will be highlighted to illustrate the procedures in applications of linear and non-linear problems. In particular, we focus on the coupled poroelasticity problem, cf. Jänicke et al. [3], as a coupled transient model problem for the numerical investigations.

REFERENCES

[1] Larsson, F., Runesson, K. and Su, F. Variationally consistent computational homogenization of transient heat flow, *Int. J. Num. Meth. Engng.*, Vol. **81**, pp. 1659–1686, (2010).

[2] Ekre F., Larsson F. and Runesson K. On error controlled numerical model reduction in FE^2-analysis of transient heat flow. *Int. J. Num. Meth. Engng.*, In Press (2019).

[3] Jänicke, R., Larsson, F., Runesson, K and Steeb H. Numerical identification of a viscoelastic substitute model for heterogeneous poroelastic media by a reduced order homogenization approach. *Comput.Methods. Appl. Mech. Engrg.*, Vol. **298**, pp. 108–120, (2016).