Fast time integration of PDEs using collocated POD and Galerkin projection on the fly

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ABSTRACT

Recently, an adaptive low-dimensional model to speed up time dependent solvers for PDEs has been proposed [1, 2, 3]. The method combines on the fly (namely, on demand) short runs of a standard numerical solver with a POD-based Galerkin system integration. Switching between both time integrators is decided on the fly by monitoring a truncation error estimate and a residual estimate (controlling mode truncation instability).

To improve the computational efficiency of the method, a crucial idea is to use a reduced number of points instead of the whole computational mesh. In this work, we propose to use the collocation strategy introduced by us in [4]. According to it, a small set of collocation points and a small set of snapshots of the snapshots matrix will be selected by applying Gauss elimination with pivoting. Then, POD is performed on the selected snapshots using an inner product based on the selected collocation points. The resultant method was called LUPOD in [4].

The performance of the synergic on the fly combination of LUPOD with Galerkin projection will be illustrated for the 1D and 2D complex Ginzburg-Landau equation with localized spatial complexity and drift.

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