# Mixed Finite Element Formulation for Lower Bounds to the Buckling Loads of Elastic Framed Structures

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## ABSTRACT

The number and variety of techniques that have been used to estimate the buckling loads of elastic structures is quite large. In a finite element context, a formulation that has demonstrated its utility to the buckling analysis of framed structures is the so-called displacement/rotation-based finite element formulation [1]. Complementary formulations, in which only force/moment quantities are taken as approximation functions, have also been proposed in the literature [2]. However, both types of formulations lead to numerical buckling loads that are upper bounds to those of the structures under consideration. Therefore, by themselves, such formulations are not entirely useful, as they do not provide information on how far the estimated buckling loads lie above the true buckling loads. In order to have a validated buckling analysis study, it is convenient to find error estimates or, ideally, to determine lower bounds to the buckling loads.

Although techniques that lead to lower bounds to the buckling loads of elastic framed structures can be found in the literature [3, 4], to the best of the Authors' knowledge, none of these techniques have been formulated in a finite element context.

We herein introduce a mixed finite element formulation for the buckling analysis of elastic framed structures which renders direct lower bounds to the buckling loads of linear elastic framed structures. This formulation is based on the approximation of the bending moments and rotation fields of the elements and relies on a Hellinger-Reissner variational principle.

The accuracy and feasibility of the formulation will be demonstrated for a set of representative numerical problems.

## REFERENCES

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