Multiscale computations based on MsFEM: model reduction and goal-oriented a posteriori error estimation

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ABSTRACT

The Multiscale Finite Element Method (MsFEM) is a Finite Element type approach for multiscale PDEs, where the basis functions used to generate the approximation space are precomputed and are specifically adapted to the problem at hand. The computation is performed in a two-stage procedure: (i) a offline stage, in which local basis functions are computed, and (ii) a online stage, in which the global problem is solved using an inexpensive Galerkin approximation. Several variants of the approach have been proposed and a priori error estimates have been established.

As for any numerical method, a crucial issue is to control the accuracy of the numerical solution provided by the MsFEM approach. In this work, we develop an a posteriori error estimate and the associated adaptive procedure. The estimate is based on the concept of Constitutive Relation Error (CRE), which we extend to the multiscale framework.

We introduce a guaranteed and fully computable a posteriori error estimate, both for the global error [1] and for the error on quantities of interest [2], using adjoint-based techniques. We discuss the accuracy of such estimates and show how they can be used to efficiently drive an adaptive discretization.

In the case of parameterized microstructures, we also investigate the additional use of model reduction techniques (such as the Proper Generalized Decomposition (PGD)) within the MsFEM approach in order to further decrease the computational costs.

REFERENCES

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[2] Chamoin, L. and Legoll, F. Goal-oriented error estimation and adaptivity in MsFEM computations. In preparation.