Towards a hexahedral hybrid equilibrium macro-element.

Edward A.W. Maunder*, José P. Moitinho de Almeida**

*College of Engineering, Mathematics, and Physical Sciences, University of Exeter,
North Park Road, Exeter, EX4 4QF, UK
email: e.a.w.maunder@exeter.ac.uk, web page: www.exeter.ac.uk

**Department of Civil Engineering, Architecture and Georesources, IST, University of Lisbon
Av. Rovisco Pais, 1096 Lisboa Codex, Portugal
email: moitinho@civil.ist.utl.pt, web page: www.tecnico.ulisboa.pt

ABSTRACT

Whilst recent advances have been made in the formulation of hybrid equilibrium elements for membrane, plate and shell problems in the context of dual analyses and error estimation, modelling of solids has proved to be more problematic. The increase in dimension from 2 for membranes to 3 for solids may be considered to be straightforward from a theoretical point of view, but in practice the implementation of the formulation brings new challenges.

One of the main features that has to be considered is the presence of spurious kinematic modes. These have been comprehensively studied for 2D plate elements (both for membrane and bending actions), and the formulation of macro-elements which effectively exclude spurious modes is well established [1]. However, the only similar macro-element known to the authors for solid problems is the tetrahedron as an assembly of 4 tetrahedra with a common internal vertex [2,3]. On the other hand it is common to exploit isoparametric hexahedral elements in conforming models.

This paper presents an outline of the formulation of a hybrid hexahedral equilibrium element. The geometry of the hexahedron assumes 12 straight edges and 6 generally flat quadrilateral faces. It is subdivided into 12 tetrahedra incident with the 8 vertices of the hexahedron and one internal vertex, the 12 edges of the hexahedron, 6 edges formed from the diagonals of the faces, and 8 internal edges. The stability of this assembly is based on the hypothesis in [1] which states that a subdivision for which each of the 8 external vertices is incident with an odd number of external edges, and the degree of the stress and displacement fields d is greater than 3, is free of spurious modes unless the faces of the hexahedron are flat. In this event, each face gives rise to $(d+1)$ local spurious modes which are only excited by discontinuous face tractions.

This crucial property is exploited in the formulation which results in a condensed element stiffness or flexibility matrix, for which the traction modes are continuous over the two triangular subdomains of each flat quadrilateral face of an element. The level of condensation can be significant with a minimal set of traction modes per face consisting of just the 6 linear modes which have non-zero resultants (forces and moments). Such an element may not be ideal for a dual analysis, nevertheless it offers the structural designer the possibility of much simpler models with a minimal total number of degrees of freedom from which to establish load paths in a solid structure. It is also shown that a flexibility method of analysis may be a practical alternative to the more conventional stiffness method.

The nature of the condensed formulation will be illustrated with a simpler potential problem in 2D.

References

[1] J.P.Moitinho de Almeida and E.A.W. Maunder, *Equilibrium Finite Element Formulations*, Wiley, 2017.

[2] J.P.Moitinho de Almeida and O.J.B. Almeida Pereira, "A set of hybrid equilibrium finite element models for the analysis of three-dimensional solids", *International Journal for Numerical Methods in Engineering*, Vol. **39**, pp. 2789-2802, (1996).

[3] O.J.B. Almeida Pereira, "Hybrid equilibrium hexahedral elements and super-elements", *Communications in Numerical Methods in Engineering*, Vol. **24**, pp. 157-165, (2008).