Randomized residual-based error estimators for parametrized equations

Kathrin Smetana*, **Oliver Zahm**† **and Anthony T. Patera**‡

* Faculty of Electrical Engineering, Mathematics & Computer Science
University of Twente
Zilverling, P.O. Box 217, 7500 AE Enschede, The Netherlands
e-mail: k.smetana@utwente.nl

† Univ. Grenoble Alpes, Inria, CNRS
Grenoble INP, LJK, 38000 Grenoble, France
e-mail: olivier.zahm@inria.fr

‡ Department of Mechanical Engineering
Massachusetts Institute of Technology
Cambridge, MA 02139, United States
e-mail: patera@mit.edu

ABSTRACT

We propose a randomized a posteriori error estimator for reduced order approximations of parametrized (partial) differential equations. The error estimator has several important properties: the effectivity is close to unity with prescribed lower and upper bounds at specified high probability; the estimator does not require the calculation of stability (coercivity, or inf-sup) constants; the online cost to evaluate the a posteriori error estimator is commensurate with the cost to find the reduced order approximation; the probabilistic bounds extend to many queries with only modest increase in cost [1]. To build this estimator, we first estimate the norm of the error with a Monte-Carlo estimator using Gaussian random vectors whose covariance is chosen according to the desired error measure, e.g. user-defined norms or quantity of interest. Then, we introduce a dual problem with random right-hand side the solution of which allows us to rewrite the error estimator in terms of the residual of the original equation. In order to have a fast-to-evaluate estimator, model order reduction methods can be used to approximate the random dual solutions. Here, we propose a greedy algorithm that is guided by a scalar quantity of interest depending on the error estimator. Numerical experiments on a multi-parametric Helmholtz problem demonstrate that this strategy yields rather low-dimensional reduced dual spaces.

REFERENCES

[1] Smetana, K. and Zahm, O. and Patera, A.T., Randomized residual-based error estimators for parametrized equations. *arXiv:1807.10489*, (2018).