A Multiscale Finite Element approach using high order polynomials

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ABSTRACT

Many applications in engineering require approximations of solutions to problems defined with heterogeneous coefficients. Using standard finite element methods (FEMs) to solve such equations can become prohibitive in terms of computational cost. Indeed the mesh discretization should match the fine scale of the heterogeneities in order to give accurate results.

Several alternatives such as the Multi-scale Finite Element Method (MsFEM for short) have been designed to address this issue. The MsFEM is a two step approach (see [1]): (i) design a coarse relevant approximation space by solving local problems and thus precompute basis functions adapted to the microstructure and (ii) approximate the solution with an inexpensive Galerkin approach upon the space built in step (i).

The method can be improved by enriching the basis by solutions to some local eigenvalue problems. This alternative has been introduced in [3] and thoroughly analysed in [2]. The approach is proven to be accurate, but it may be too costly, due to the resolution of *eigenvalue* problems.

We present here a similar method which enriches the MsFEM basis differently: we add solutions to local problems with polynomials of high degree as boundary conditions to the standard MsFEM basis set. The use of polynomials allows us to show convergence results thanks to their versatile approximation properties at reasonable computational cost. Some numerical experiments are shown to illustrate the effectiveness of the method.

REFERENCES

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