Transport maps sampling with PGD model reduction for real-time Bayesian data assimilation

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ABSTRACT

The work is placed in the framework of the Dynamic Data Driven Application System (DDDAS) concept, in which a strong and real-time interaction between numerical models and physical systems is required. In particular, the model constantly needs to precisely describe the evolving system, so that in-situ measurements have to be used sequentially in order to update model parameters in real-time. The motivation of this work is thus to address sequential and fast inference of model parameters in a full Bayesian formulation [1], in which parameters to be identified are considered as random variables and the result of the inference problem is the posterior Probability Density Function (PDF) on these parameters. Bayesian inference is a natural and convenient method to consider uncertainties such as measurement error or modeling error, as these uncertainties are naturally propagated through the model. Nevertheless, it is associated with costly computations. Indeed, the inverse approach requires solving numerical models described by PDEs for all combinations of the parameters. Furthermore, the PDFs have to be explored to derive useful information such as mean, standard deviation, maximum, or marginals, which requires the computation of multi-dimensional integrals over the parametric space. Consequently, the use of full Bayesian inference remains nowadays intractable in most real-time applications.

The new approach we propose is twofold. First, the Proper Generalized Decomposition (PGD) [2] is used to reduce the computation time for the evaluation of the posterior density in the online phase. PGD builds a multi-parametric solution in a offline phase and leads to cost effective evaluation of the numerical model depending on parameters in the online inversion phase. It was already used in the context of Bayesian inference [3]. Second, and as an alternative to classical Markov Chain Monte-Carlo (MCMC), we refer to the promising Transport Maps sampling technique [4]. It builds a deterministic map between a reference probability measure and the posterior measure resulting from Bayesian inference. This way, all integrals according to the posterior measure are transported according to the reference measure. The determination of the map involves the solution of a minimization problem which is facilitated by a PGD solution (the straightforward gradient and Hessian information enables an important speed-up in the procedure). Eventually, Uncertainty Quantification on outputs of interest may be performed owing to the PGD model. Several numerical examples will highlight the performance of the proposed method.

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