Simplified closed-form expressions for horizontal reactions in linear elastic arches under self-weight

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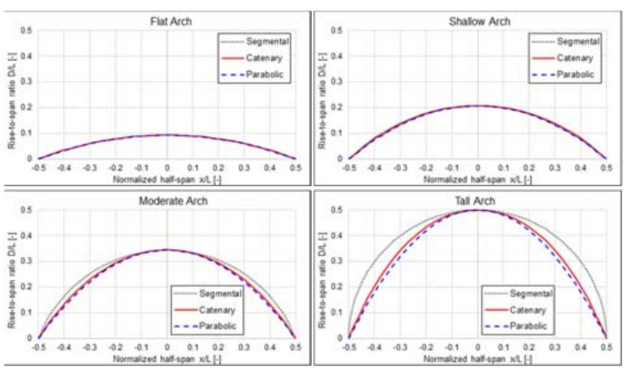
Abstract

Since ancient times, symmetric arches have been frequently used as the main structural members in bridges and buildings. In the past, builders mostly used segmental arches, i.e., those with the shape of an arc of circle, while in the modern times parabolic shape is also frequently used. Finite element modeling and discrete element modeling are typical modern tools for determining reactions and internal forces in arches. However, while these tools are excellent in performing detailed analysis, they might be less suitable for conceptual design and "back-of-the-envelope calculation" due to their complexity of application. Simplified closed-form formulae for reactions and internal forces might be more appropriate as they enable fast solutions and easy development of parametric studies. Hence, the aim of this paper is to present novel simplified closed-form expressions for determination of horizontal reactions in segmental, catenary, and parabolic arches under self-weight (see Figure 1). Three typical structural systems for symmetric linear-elastic arches with constant cross-section and various rise-to-span ratios were considered: three-hinged, two-hinged, and hingeless. It was found that the horizontal reactions practically do not depend on arch shape or number of hinges, but only on linear weight, span, and equivalent angle of embrace (see Equation 1 [15]). Surprisingly, with few exceptions, this single formula provides almost universal solution for all shapes of arches, rise-to-span ratios, and boundary conditions.



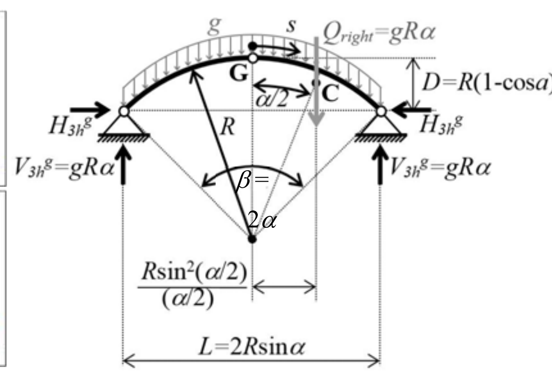


Figure 1: Left: four characteristic rise-to-span ratios for segmental, catenary, and parabolic arch; Right: loading and geometry of typical segmental arch (three-hinged arch show in figure) [15].

$$H = \frac{gL}{\beta} \tag{1}$$

In addition, linear relationship between horizontal reactions of the three structural systems (hingeless, three-hinged, and two-hinged) was discovered, which is valid for any shape (segmental, parabolic, and catenary) and any rise-to-span ratio (see Equation 2 [15]), regardless whether Equation 1 applies or not.

$$H_{three-hinged} \approx (H_{two-hinged} + H_{hingeless})/2 \tag{2}$$

References

[15] B. Glisic, "New in old: simplified closed-form equations for linear-elastic segmental, catenary and parabolic arches, and some new discoveries regarding their structural behaviors," *Acta Mechanica*.