

# The formfinding of a building of unlimited height

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## Abstract

Greenhill [1] was the first to obtain formulae for the buckling due to self-weight of columns of various shapes, including prisms and cones. But it was not until 1966 that Keller & Niordson [2] found the optimum profile for a column. Both these papers assumed a material of unlimited strength and in this paper we combine the effect of buckling with the fact that the vertical stress has to be limited.

If the cross-sectional dimensions of a column increase exponentially as one progresses downwards from the top, then the compressive stress due to self-weight can be made to approach a constant value. The same applies to the compressive stress in the columns and walls of a building. Thus, there is no limit to the height of a building based upon stress, if one is prepared to sufficiently increase the dimensions of the vertical structure.

The usual approach to buckling is to find the buckling load and mode shape for a given structure. However, in this paper we reverse this process and instead specify the buckling mode shape and use this to formfind the vertical profile of a column or building of unlimited height. This involves the integration of a second order differential equation. The reason for interchanging the known and unknown is that we have a boundary condition at the base of the column for zero rotation due to buckling, but no condition for the building profile.

We find that the profile of the very top of the building is controlled by wind load. Lower down buckling controls the profile and lower down still it is the limit on vertical stress. The diagram on the right shows the building profile and the buckling mode is shown in red.

## Keywords

Tallest building, buckling under own weight, optimization.

## References

- [1] Alfred George Greenhill, Determination of the greatest height consistent with stability that a vertical pole or mast can be made, and of the greatest height to which a tree of given proportions can grow, *Proceedings of the Cambridge Philosophical Society, Mathematical and Physical Sciences*, vol. IV, Number II, pp. 65-73, 1881.
- [2] Joseph B. Keller and Frithiof I. Niordson, The Tallest Column, *Journal of Mathematics and Mechanics*, vol. 16, pp. 433-446, 1966.

