UNCERTAINTY ANALYSIS OF MATHEMATICAL MODELS AND DEEP NEURAL NETWORKS

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Mathematical models used in scientific and engineering practice often contain some uncertainty. It can arise due to inaccuracies in physical data, geometric character- istics, boundary and initial conditions and many other reasons. Uncertainty of the statement entails limitations on the accuracy of the result. In this case, we have a ”cloud” of possible solutions instead of the unique solution defined by the fully known data. As a result an extreme precision arises that limits any quantitative analysis method (we can identify it with the diameter $d(S)$ of the solution cloud S). This quantity is principally important and methods for determining it have been studied in a number of publications. In [1], the theory of a posteriori estimates of functional type was used to determine the limiting accuracy of mathematical models based on partial differential equations. It yields two–sided bounds of $d(S)$. For relatively simple linear problems, these bounds are known and can be directly computed. For more complicated problems (e.g., for those described by nonlinear PDEs) they may be coarse and computationally difficult. Therefore, in addition to the analytical way of the uncertainty estimation we consider a method based on deep learning of neural networks, which analyse the data and give a realistic value of $d(S)$ (i.e., they are constructed to solve an ”uncertainty recognition” problem). In the talk, we discuss inexpensive methods able to generate sufficiently large sets of teaching examples and compare the results obtained by networks and analytical methods.

References

[1] Mali, O., Neittaanmäki, P., Repin S. 1998, Accuracy Verification Methods. Theory and Algorithms. Springer, Berlin.