Stabilization method for improving the condition number of the matrix of system of equations under the Cartesian Grid Finite Element Method

E. Nadal, M. Tur, J.M. Navarro, J. Albelda and J.J. Ródenas

Research Center on Mechanical Engineering (CIIM)
Universitat Politècnica de València
Building 5E, 46021 Valencia, Spain
e-mail: ennaso@upvnet.upv.es, matuva@mcm.upv.es, jonaji@upv.es, {jalbelda, jjrodena}@mcm.upv.es

ABSTRACT

When using methods under the umbrella of Fictitious Domain such as the Cartesian Grid Finite Element Method (cgFEM) [1] to solve the linear elastic problem, some nodes of the mesh fall outside of the physical domain (pathological nodes), having a low associated stiffness in comparison with those located within the physical domain, as can be seen in Figure 1. This issue, typical of Fictitious Domain methods, leads to a deficient condition number of the matrix of the system of equations, making difficult the use of iterative solvers for large problems. Some authors proposes the modification of the Cartesian mesh structure [2] by forming aggregates while others propose the use of implicit domain extensions [3]. In this work, in order to preserve the Cartesian structure of the mesh, a new formulation for the stabilization problem is proposed. The formulation consist of penalizing the free movement of those nodes by an smooth extension of the solution of the interior of the domain, by means of a post-process of the solution via a displacement recovery technique. Graphically the proposed stabilization can be considered as an spring that anchors the node and a force that compensates the reaction of the spring, as shown in Figure 1, thus keeping the convergence properties of the problem. The numerical results show an improvement of the condition number, a decreasing in the number of iterations of the iterative solver while preserving the problem accuracy.

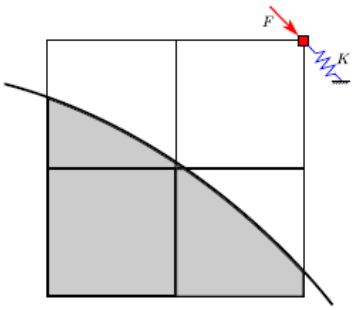

Figure 1: Example of a pathological node (red) with the proposed stabilization scheme.

Keywords: cgFEM, Iterative Solver, Fictitious Domain, Condition number.

Acknowledgments: With the support of Ministerio de Economía, Industria y Competitividad of Spain (DPI2017-89816-R) and the Generalitat Valenciana (PROMETEO/2016/007).

REFERENCES

[1] E Nadal, JJ Rdenas, J Albelda, M Tur, JE Tarancón, FJ Fuenmayor. Efficient Finite Element Methodology Based on Cartesian Grids: Application to Structural Shape Optimization. Abstract and Applied Analysis 2013, Article ID 953786, 19 pages (2013).

[2] S Badia, F Verdugo, A.F. Martn. The aggregated unfitted finite element method for elliptic problems. Computer Methods in Applied Mechanics and Engineering. 336:533-553, 2017.

[3] E Burman. La pénalisation fantome. Comptes Rendus Mathematique. 348(21-22):12171220, 2010.