A PGD-HDG methodology for the simulation of incompressible flows in parameterized geometries

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ABSTRACT

Nowadays, the main computational bottleneck in computer-assisted industrial design procedures is represented by the necessity of testing a large number of parameter settings (e.g. material properties, boundary conditions, geometries) for the same problem. This leads to a significant increase of the computational cost of such procedures. Moreover, the well known curse of dimensionality limits the possibility of performing parametric analyses using a mesh-based approach to few parameters, making this strategy unaffordable for realistic problems. This issue can be circumvented by means of a reduced order modelling framework. In this work, a methodology that combines the proper generalized decomposition (PGD) [1] and the high-order hybridizable discontinuous Galerkin (HDG) method [2] is proposed for the solution of incompressible flow problems in domains with parameterized geometries. A parametric description of the domain is considered to map a family of physical configurations of interest to a fixed reference domain on which the computation is performed [3, 4]. PGD is exploited to construct a computational vademecum of the solution in which the parameters describing the geometry of the domain act as extra-coordinates of the problem. The formulation of HDG as a mixed method allows the PGD to perform an exact separation of the terms introduced by the parametric mapping into products of functions depending either on the spatial or on the parametric unknowns. Thus, the computationally demanding preprocessing step discussed in [3, 4] to construct an approximation of such separation is circumvented by the combined PGD-HDG strategy. Moreover, HDG offers a flexible and efficient framework to devise high-order and degree adaptive discretizations, especially suitable in the context of flow simulations. Convergence results will be discussed to validate the approach and more realistic test cases involving incompressible flows in variable geometries will show the potential of the proposed PGD-HDG methodology to handle problems of interactive design in an industrial context.

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