A goal-oriented error estimator for quasi-static poro-elasticity with application to computational homogenization

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ABSTRACT

Multiscale methods are of high interest in the engineering community due to their ability to predict the overall response at a suitable macroscale, while accounting for the structure on the underlying scales. One standard approach is the so-called "finite element squared" (FE^2) procedure, where a new boundary value problem is defined on a Representative Volume Element (RVE) in each quadrature point of the (macroscale) mesh. The RVE problem is solved in order to obtain the effective response of the material and is thus replacing the otherwise needed macroscale constitutive relation of empiric character. It is known that the FE^2 strategy can be computationally intractable, in particular for fine macroscale meshes in three dimensions. Therefore it is of interest to reduce the cost of solving the individual RVE problems by introducing a reduced basis, here denoted Numerical Model Reduction (NMR). It is obvious that the richness of the reduced basis will determine the accuracy of the solution, which calls for error control. Several methods for estimating the error for a reduced basis approximation have been developed, one example for a linear heat-flow problem with a spectral basis is Aggestam et al. [1].

Jänicke et al. [2] considered homogenization of quasi-static poroelasticity with application to seismic attenuation. It was demonstrated that the apparent macroscale properties pertain to viscoelasticity. Proper Orthogonal Decomposition (POD) was used to find a set of pressure modes that defined the reduced problem, and the efficiency of the procedure was demonstrated.

In this presentation we present a strategy for estimating the error due to NMR, with application to homogenization of quasi-static porous media, thus extending the work by Jänicke et al. [2]. An estimator of the "NMR"-error is introduced, meaning that we ignore the error from time- and space-discretizations by considering the fully resolved finite element problem to be the exact one. In particular we aim for guaranteed explicit bounds on the error both in (i) energy norm and (ii) "quantities of interest", within the realm of goal oriented error estimation. Numerical investigations consider a single RVE problem in three dimensions.

REFERENCES

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