Explicit time integrators and forward-in-time goal oriented adaptivity

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ABSTRACT

Goal-oriented adaptive algorithms for partial differential equations seek to reduce the error of the solution in some quantity of interest. In time dependent problems, we employ a full space-time variational formulation to represent such error in the quantity of interest as an integral over the whole space-time domain [1]. The adaptive process involves solving a dual problem that runs backwards in time and that it is computationally expensive. Most of the existing goal-oriented algorithms employ implicit methods in time to solve both primal and dual problems as it is known that they can be reinterpreted as Galerkin methods [2]. In this work, we present two different strategies to perform goal-oriented adaptivity for time-dependent parabolic problems that are cheaper than classical methods.

First, we propose an adaptive algorithm that solves the primal and dual problems employing explicit methods in time. For that, we consider a variational formulation of the explicit Runge-Kutta methods [4]. We derive an appropriate error representation and propose a goal-oriented adaptive algorithm in space. In time, we perform the adaptivity by imposing the Courant-Friedrichs-Lewy (CFL) condition to ensure the stability of the method [3].

The second proposed algorithm employs a pseudo-dual problem that runs forward in time. This leads to a forward-in-time adaptive algorithm that works for some specific situations. However, it is not possible in general to define a dual problem running forwards in time that provides information about future states. As an alternative, we propose a mixed algorithm that combines both classical and the proposed pseudo-dual problems. We provide some numerical results in 1D space + time for the diffusion and advection-diffusion equations to show the performance of the proposed explicit-in-time and forward-in time goal-oriented adaptive algorithms.

Keywords: linear advection-diffusion equation, goal-oriented adaptivity, explicit methods in time, pseudo-dual problem, error representation, finite element method

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